

# Quantum Wave Behavior in the Prescence of Finite Potential Wells

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The phenomenon of redshift plays an essential role in our understanding of the universe. The redshift or increasing wavelength of light observed from distant celestial objects is attributed to the recessional velocity of distant astronomical objects due to space-time expansion. As observational techniques improve, however, there is room for alternative frameworks that might offer complementary or refined interpretations.

In the following sections, we consider resonant transmission with respect to a potential well and, then, building upon this foundation, we link redshift phenomenon with a  $g$ -factor (a dimensionless quantity associated with quantum systems and their magnetic properties) and temperature. We consider how temperature dependent variations in a  $g$ -factor affect the thermodynamic properties of

cosmological systems and their observational signatures.

By examining how these temperature dependent variations influence observed spectral shifts, this framework tries to address key questions about the relationship between local physical parameters and the observed large-scale structure of the universe.

This work wants to provide a different perspective that we might use to gain a better understanding of the redshift, temperature, and intrinsic material properties. The following outlines the theoretical basis of our model, derives important equations, and compares predictions with observed data [1].

We begin with an examination of the transmission coefficient, derived from the conditions of a finite potential well (or a finite square well) which tells us how likely it is that a particle will be transmitted beyond the barrier or out of the well.

So, we hope to show that photons comprising a phase velocity are transmitted out of a potential well—and reabsorbed by a blackbody.

The potential  $V(x)$  of a finite square well is defined as:

$$V(x)=$$

$$\begin{cases} 0 & \text{for } x < -a \text{ (outside the well, left),} \\ -V_0 & \text{for } -a \leq x \leq a \text{ (inside the well),} \\ 0 & \text{for } x > a \text{ (outside the well, right),} \end{cases}$$

where:

$-a$  to  $a$  is the width of the well.  
 $V_0 > 0$  is the depth of the well.

Resonant transmission means there is no reflection: The transmission probability, then, means that  $T = 1$ . The formula for a wave passing above and or through a potential is:

$$\frac{1}{T} = 1 + \frac{1}{4} \frac{V_0^2}{E(E+V_0)} \sin^2(2k_2 a) \text{ where } k_2 \text{ is the}$$

wave number of the regions of and above the well  
and  $a$  is a measure of distance.<sup>1</sup>

We have:  $k_2^2 = \frac{2m(E+V_0)}{\hbar^2}$  from the Schrodinger

equation, which is:

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = \left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x, t) \right] \Psi(x, t) .$$

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<sup>1</sup> If you want to see the complete and laborious derivation of this equation (for  $T$ , the transmission coefficient), you can tell a free AI assistant, such as ChatGPT, to take you through it step by step.

Note that (from our transmission equation) where  $E$  goes to 0, the second term goes to infinity and therefore  $T$  is going to zero and there will be no transmission.

If  $E$  goes to infinity, the second term goes to zero and you get  $T = 1$ .

We'll re-write the transmission equation in unit free language (where  $V_0$  is understood to be negative): then, where

$k_2^2 = \frac{2m(E+V_0)}{\hbar^2}$  and

$2k_2a = 2\sqrt{\frac{2ma^2(E+V_0)}{\hbar^2}}$  : (we put the  $a$  into the

square root); then we have:  $2\sqrt{\frac{2ma^2V_0\left(1+\frac{E}{V_0}\right)}{\hbar^2}}$ .

Now, we define a unit free energy to be  $e \equiv \frac{E}{V_0}$  [2].

This  $e$  compares the energy of the energy eigenstate (an eigenvector of some linear operator and or an energy vector and or energy with a directional flow) to the depth of the potential.

The above includes  $z_0^2$  which is  $\frac{2ma^2V_0}{\hbar^2}$

which is a unit free number associated with a potential well that tells us how deep or profound your potential is; we can rewrite the argument of

the sine function  $2k_2a$  as  $2z_0\sqrt{1+e}$  .

Then we had  $\frac{V_0^2}{E(E+V_0)}$  . We can divide the denominator by  $V_0^2$  if we divide the numerator by

$V_0^2$  to get  $\frac{E}{V_0} \left(1 + \frac{E}{V_0}\right)$  : then we have:

$$\frac{1}{T} = 1 + \frac{1}{4e(1+e)} \sin^2(2z_0\sqrt{1+e}) .$$

For perfect transmission (no reflection) we need the argument of the sine function to be equal to multiples of  $\pi$  (where the function is equal to ze-

ro). Then:  $2z_0\sqrt{1+e} = n\pi$  . The energy,  $e$  ,

would be positive, so the smallest  $2z_0\sqrt{1+e}$  can

be is  $2z_0$  . So,  $n$  must be greater than or equal

to  $\frac{2z_0}{\pi}$  . Then  $4z_0^2(1+e_n) = n^2\pi^2$  .

Then  $e_n = -1 + \frac{n^2\pi^2}{4z_0^2}$  ; then, where  $e = \frac{E}{V_0}$  , we

multiply by  $V_0$  to get back to the original units:

$$E = -V_0 + \frac{n^2\pi^2 V_0}{4 \cdot \frac{ma^2 V_0}{\hbar^2}} \text{ and so:}$$

$$E = -V_0 + \frac{n^2\pi^2 \hbar^2}{2\bar{m}(2a)^2} .$$

Now, the total energy  $E$  includes  $-V_0$  , and  $E$  is negative when solving for a bound state. Therefore the total energy is  $E_n - V_0$  ,

where  $E_n = \frac{n^2\pi^2 \hbar^2}{2\bar{m}(2a)^2}$  (the kinetic energy of the particle) and where  $2a$  is the width

of the well—(from  $-a$  to  $a$  ).

We'll argue, now, that potential wells transmit photons at resonance.

From  $\frac{1}{T} = 1 + \frac{1}{4} \frac{V_0^2}{E(E+V_0)} \sin^2(2k_2 a)$  , we look at

$2k_2 a$  : the condition was that  $k_2(2a) = n\pi$  :

then we have  $\frac{2\pi}{\lambda}(2a) = n\pi$  and so  $\frac{2a}{\lambda} = \frac{n}{2}$  .

So, the wavelength fits into  $2a$  a half-integer number of times. That's what you have in an infinite square well; if you have a width you could have a half a wavelength.

For  $n = 1$ , we have a half wavelength (in the square well); for  $n = 2$ , we have one wavelength; for  $n = 3$ , we have three halves: you get halves and halves increasing.

In the well-region of half waves the resonance allows the wave (the wave above or the wave that would be above the well) to be completely transmitted. Resonance (complete transmission) is due to having an exact number of halves in the region.

Now we want to argue that a  $g$ -factor, a dimensionless constant (such as an extradimensional angle), multiplied by the temperature of the energy in our well will yield a change in temperature we are looking for.

So a  $g$ -factor—a dimensionless constant that is a function of the magnetic moment of a particle—will account, in part, for a difference between the temperature of the well and the temperature of the cosmic microwave background radiation (the CMB).<sup>2</sup>

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<sup>2</sup> The lesser the  $g$ -factor, the greater the temperature would be; higher temperatures make it more difficult for magnetic moments and magnetic fields to align.

## 2

In this section we examine how the temperature in a potential energy well is affected by a  $g$ -factor—and resonant transmission. This argument helps us to show a relationship between the well's temperature and the CMB. When we study the math governing spin magnetic moments, angular momentum, and certain quantum constants, we will show how temperature variations are quantitatively determined (they depend on photons passing through)—allowing us to develop our understanding of energy distributions throughout the cosmos. We offer a bridge between quantum mechanics and macroscopic thermodynamic observations; we'll begin, then, with a derivation of a  $g$ -factor, and then we'll use that to discuss energy transfer and blackbody interactions—such that we can study redshift phenomena in a different way.

We have the equation:  $\mu = g * \frac{e}{2m} * S$  .



That's where  $(\mu)$   $\mu$  is the spin magnetic moment, where the magnetic moment is the strength and orientation of a magnet or object that exerts a magnetic field.

It determines the magnitude of the torque the object experiences in a magnetic field.

Then  $S$  is the spin angular momentum which tells us about the internal motion of an object: it tells us how fast an object would be spinning and how hard it is to make it spin.

The elementary charge is  $e$ ,  $g$  is the  $g$ -factor, and  $m$  is the mass.

Now, photons have spins of either positive one or negative one, so  $S$  is  $\frac{\hbar}{2}$  as opposed to  $\frac{\hbar}{1}$ .

(Photons have spin, and they are a function of an electric field, so they, albeit indirectly, generate a magnetic field, which, in turn, generates an electric field).

So, a photon propagating in the  $z$ -direction with an angular frequency  $\omega$  carries a magnetic

moment of  $m_z = \pm \frac{ec}{\omega}$  along the propagation direction [3].

The  $\pm$  signs stand for right hand and left hand circular helicity.

We might think of light as generating a charge and or an electric field based on the motion of the magnetic moment of the photon—or as a wave propagating through an electric charge intrinsic to space; either way we must arrive at the conclusion that the magnetic moment of the photon is equal to the above—or the electric field generated by the magnetic field of the photon is equal to the magnetic moment times omega.

We can solve for the  $g$ -factor of a photon by setting the two equations for the magnetic moment equal to each other: so  $m_z$  becomes a photon traveling in two directions per second, like a spiral or helix—which would be the direction of the electric and magnetic field times the direction(s) of the angular momentum per second.

So, we have:  $\frac{ec}{\omega} * v = g * \frac{e\hbar}{2m}$  so  $g =$

$$\frac{ecv2m}{e\hbar\omega} .$$

Photons have no mass, so we must solve for the mass in terms of the wavenumber.

At an energy level of  $299792458^2$  , and a velocity of  $299792458$  , we have  $m$  in terms of  $k$  to be  $4.420841291 * 10^{-28}$  .

I got that from the following:  $m = \frac{k\hbar}{v} = \frac{KE^2}{v^2}$  .

So the change in  $KE$  will correspond to a change in  $k$  that will yield  $m$  in terms of  $k$  . Thus, we're transforming a mass into an *effective* wave number that accounts for its relationship with the energy or forces around it.<sup>3</sup>

(Thus, the photon  $g$ -factor depends on the energy level of the system—in this case a bound state with an energy level of  $299792458^2$  ).

Hence, we solve for  $n$  by taking the mass of the system to be equal to two. (That's where  $E_n =$

$$\frac{\hbar^2 n^2 \pi^2}{2ma^2} \text{ where } a = 2\pi \text{ ).}$$

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<sup>3</sup> Terms like effective mass and effective wave number relate the mass and or the wave number with respect to outside conditions—such as a negative potential. These effective properties are not intrinsic to the particle—they represent the particle's reaction to its surroundings.

Then  $\Delta k$  is given by  $\frac{n}{\Delta n}$  where  $\Delta n$  is given by  $\log_2 n = \Delta n$  .

(The change in  $n$  is a function of quantized units of  $n\pi$  where everything is bisected such that every change in  $n$  has a base of two).

(This also means that the change in  $k$  is given by an accumulation of bisections—as  $n$  increases, each value for  $a$  is halved or bisected).

Then  $m_2$  would be  $\frac{\Delta k \hbar}{v}$  .

We hold the energy of the well constant and we have  $KE = \frac{1}{2} m_2 c^2 = 1.256751656 * 10^{15}$  .

Then we have  $\Delta KE = 299792458^2$  divided by that.

Then  $m$  in terms of  $k$  is  $\frac{\Delta KE 2}{v \hbar} = \bar{k} = 4.524025787 * 10^{27}$  .

Then, where the system is a function of kilograms we have  $x \text{ kg} = \frac{x \text{ kg}}{\bar{k} \text{ m}^{-1}}$  .<sup>4</sup>

So, where  $m = \frac{k \hbar}{v} = \frac{KE 2}{v^2}$  , the mass was equal to

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<sup>4</sup> Then the kilogram units cancel with the wavenumber units.

two so  $m$  in terms of  $k$  became  $4.420841291 * 10^{-28}$  .

Now, when the mass is undefined we take the general form of  $m$  in terms of  $k$  to be  $4.524025787 * 10^{27}$  .

Therefore  $\omega = 1.356268811 * 10^{36}$  .

So, the photon  $g$ -factor is:  $g =$

$$\frac{c * 4.420841291 * 10^{-28} * 299792458 \frac{m}{s}}{4.524025787 * 10^{27} * 1.054571817 * 10^{-34} * \omega} =$$

$$6.410437602 * 10^{-41} .$$

We solve for the temperature of the  $299792458^2$  joules in the well using Boltzmann's constant:  $E = k_B T$  where  $k_B = 1.380649 * 10^{-23} \frac{J}{K}$  . That is  $6.509657261 * 10^{39}$  .

We multiply by our  $g$ -factor (which, as a function of  $\frac{cm}{m^2 s^{-1}}$  , or an amount of light per area,

scales the temperature) and we have a temperature of .399721442 which corresponds to a wavelength of .00725 .

We subtract from the temperature of the CMB and we have: 2.325778558  $K$  .

That's the temperature lost by the well and absorbed by a blackbody:<sup>5</sup> so the blackbody temperature, or the remaining temperature, would be the temperature of the CMB (2.7255) divided by the above after transmitting this energy to the well; that's the number of times this energy can go into the blackbody and be passed on to the well: so there's extra energy in the blackbody, such that the blackbody is radiating outward as it should be).

That would leave the blackbody with a temperature of 1.171865649 which corresponds to a wavelength of .002473 .

Now,  $V_0$  has to be greater than  $E_n$  or the particle wouldn't be trapped inside the well.

Therefore the total energy  $E$  must be greater than  $E_n$  or the particle will be ejected.

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<sup>5</sup> A blackbody would absorb all the radiation around it—and emit radiation at a rate based on its temperature. The CMB is a form of blackbody radiation and has a temperature of 2.7255  $K$ .

Solving for  $k$  using the equation  $E =$

$$\frac{\hbar^2 k^2}{2m} \text{ where } 2m \text{ would be } 2 * 4.420841291 * 10^{-28}$$

we have  $k = 5.977189214 * 10^{28}$  and

$$\omega = 1.791916246 * 10^{37} .$$

$$\text{Then } E = \hbar\omega = 1.889703213 * 10^3 .$$

Then we have  $E = -V_0 + E_n$  which means that

our photon is ejected: (the total energy of the particle,  $E$ , is negative when  $V_0 > E_n$  and that would yield a bound state).

In our case  $E$  is positive, so the particle would be ejected from the well.

Back to our equation:

$$\frac{1}{T} = 1 + \frac{1}{4} * \frac{(V_0)^2}{E(E+V_0)} * \sin^2(2k_2 a) .$$

Now,  $T$  tells us the probability that a particle will be transmitted beyond the range of the well: then  $1 - \frac{1}{T}$  tells us the probability that a particle will be reflected.

If we change  $a$  without changing  $k$  then  $ka$  won't equal  $n\pi$  and the particle won't be bound. (The sine function must be equal to 0 since the energy outside the well is equal to 0. That leaves us with  $n$  states inside the well).

If we let  $a$  (the breadth of the well) be equal to the peak wavelength of the remaining temperature in our well, then the probability that a particle will enter the well (from normal space) is equal to

$$\frac{1}{1.203449214 \times 10^{11}} \cdot \text{(We plugged .00725 in for } a \text{ in}$$

the above equation and used  $E$  and  $E_n$  to solve for  $V_0$  ).

Thus, the well must absorb a photon to replace the one that just left, (since it's temperature is that much less than the surrounding space).

The only place that photon can come from is a blackbody—or there would be no well, (since the energy outside the well must be zero plus whatever photon(s) the well just emitted).

Hence, one minus the above is the probability that a particle will be reflected.<sup>6</sup> But it can't be reflected

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<sup>6</sup> A transmitted photon would escape the well—and a reflected photon would remain in the well (for a limited amount of time).



since the well is allowing particle(s) to escape—and it can't likely come from the outside. So the next photon must come from a blackbody (the only source of energy that can reach the well in its colder state).

So we have the temperature of the well (with an energy of  $299792458^2$  ) to be  $6.509657261 * 10^{39}$  multiplied by our  $g$ -factor (  $6.140437602 * 10^{-41}$  ).

That left us with a temperature of  $.399721442$  and, thus, a wavelength of  $.00725$  .

Next, we solved for the temperature lost by the well when it ejected a particle: that was the temperature of the CMB minus the temperature above: we have  $2.325778558$  which corresponds to a wavelength of  $.001246011$  .

Then we divided the temperature of the CMB by the temperature lost by the well to arrive at the temperature left to the blackbody: we have:  $1.171865649$  which corresponds to a wavelength of  $.002473196$  .

Now, using Wien's law, (and measuring per unit wavelength): we input the temperature of the CMB and we get the wavelength:

.001063092 ; that was  $\lambda = \frac{b}{T}$  where  $b =$

$2.897771955 * 10^{-3}$  .

Then we showed that the CMB, minus the temperature of our well times the  $g$ -factor, left us with a wavelength of .001246011.

If we divide that wavelength by the peak wavelength of the CMB (.001063092) we have the change in wavelength from the bottom of the well (.001246011) to the top of the well (.001063092) caused by the energy being more negative at the bottom of the well.

That's 1.172063189 . We multiply that by the wavelength left to the blackbody (.002473196) and we have .002898742 . That's the wavelength gained by the blackbody for a well with a kinetic energy level of  $299792458^2$  .

We know, however, that the temperature of the CMB is 2.7255—so the wavelength lost by the blackbody must be:

.002898742—.001063092=.001835650 .

So, the blackbody is radiating at that wavelength notwithstanding what goes into the well.

Now, if we measure the peak frequency using Wien's frequency law and convert that to wavelength using the equation  $c = \nu\lambda$  (where  $\nu$  is the frequency) we have a different wavelength for the CMB: .00185171376.

(Wien's frequency law:  $\nu_{peak} = \frac{kT}{h}$  where  $k$  is the Boltzmann constant,  $T$  is the temperature, and  $h$  is Planck's constant.)

So, where the black body is transmitting a wavelength of .001246011 to the well, we showed that the black body would be left (after accounting for the loss in energy from the bottom of the well to the top of the well and, also, the temperature gained by the blackbody from the CMB), with a wavelength of .0018355650 .

Hence, the peak wavelength of the CMB is determined by the temperature of the black body radiation and the temperature of the phase velocity—the temperature lost by the well when it ejected a particle).

Finally, we show that the CMB is what it is because the magnetic moment of the particle in

question, the function of our  $g$ -factor, is proportional to an angular momentum (such as spin angular momentum) which, in turn, would travel slower than the speed of light—or stretch out the wavelength of the radiation in question as the particle pulls itself free of the influence of the well.

## 3

Now we'll solve for a  $g$ -factor where  $k$  is equal to visible light—so we'll take  $k$  to be  $8.975979010 * 10^6$  .

Then we have  $\hbar ck$  to be  $2.837779794 * 10^{-19}$  and we'll take half of that to be  $KE$  .

Then, we have  $m = \frac{2KE}{v^2} = 3.157455847 * 10^{-36}$  .

We solve for  $n$  and we have:  $7.161795070 * 10^6$  .

Then  $\Delta k = \frac{n}{\log_2 n} = 3.553296294 * 10^5$  .

Then  $m_2 = \frac{\Delta k \hbar}{v} = 1.249933422 * 10^{-37}$  .

So the new  $KE$  must be  $5.616920681 * 10^{-21}$  .

Therefore,  $\Delta KE$  must be  $\frac{\frac{1}{2} * 2.837779794 * 10^{-19}}{5.616920681 * 10^{-21}} = 25.260992235$  .

Then  $m$  in terms of  $k$  is  $\frac{\Delta KE^2}{v\hbar} = 1.598024882 * 10^{27}$  .

(I got that from  $m = \frac{\hbar k}{v} = \frac{\Delta KE^2}{v^2}$  .)

Then we have:  $g = \frac{ec^2(\bar{m})}{e\hbar\omega}$  and solving such that mass  $\bar{m}$  is given in terms of  $k$  we have:

$$\frac{c * c * \left( \frac{3.157455847 * 10^{-36}}{1.598024882 * 10^{27}} \right)}{1.598024882 * 10^{27} * 1.054571817 * 10^{-34} * 4.7900760 * 10^{35}} .$$

So  $g = 2.199535836 * 10^{-75}$  .

We divide  $KE$  by Boltzmann's constant and we have the temperature in the well to be:  $1.0276997769 * 10^4$  .

We multiply  $g$  times  $T$  ; then we subtract that very small number from the temperature of the CMB (2.7255) and we can see, then, that well temperatures corresponding to visible light hardly affect the CMB.

So, if a star's light is redshifted, then it must be generating scattered states that become bound states that increase the potential in the well.

Finally, if  $E_n = 299792458^4$  , then, after going through the process above, and arriving at a temperature of  $1.245901619 * 10^{33}$  , we subtract that from 2.7255 , and we can see that we end up with increasingly red-shifted light.

So, well temperatures would be negative; then the particle trapped in a bound state would escape, turn into a scattered state, and, in so doing, add to the temperature outside the well.

Thus, as the energy associated with an expanding universe increases, we approach the temperature of the CMB—meaning that the further away we get, the greater the energy must have been; so, we could be that much closer to the Big Bang, or we could be dealing with an increasingly deep potential—one that, nevertheless, maxes out at a temperature of 2.7255—meaning that the photon travels at a greater energy level from the bottom of the well, and decelerates at the same rate that it loses energy when it is transmitted from the well.

This study explores the relationship between quantum mechanics and thermodynamics by analyzing the behavior of particles in finite potential wells. We study resonant transmission with respect to a  $g$ -factor and relate that to temperature variations within a potential well. We detail how local quantum behavior can influence largescale cosmological observations—leaving us with a new perspective on red-shift phenomena and blackbody radiation. These findings connect theoretical models with empirical data: we conclude that the temperature dependent nature of energy distributions throughout the universe could explain the behavior of the CMB without totally relying on a Big Bang to explain it. This work offers a framework for further study—for example: if the universe is expanding, where did this energy (that made it so) come from? Moreover, does the acceleration of universal expansion relate to the acceleration of a photon as it is transmitted from the forbidden region (the blackbody region) to the light wave it, and the photons that come after it, define?



## References

[1] I used an AI assistant (ChatGPT) to help with my introduction(s) and my conclusion.

[2] B. Zwiebach, *Quantum Physics I*, MIT OpenCourseWare, Spring 2016,  
<https://ocw.mit.edu/courses/physics/8-04-quantum-mechanics-i-spring-2016/>.

Note: I often rely on my lecture notes from the above class when setting up a problem or laying a foundation for my original ideas.

[3] Z. Saglam and G. Sahin, J. Mod. Phys. 6, 937 (2015). <https://doi.org/10.4236/jmp.2015.67098>.